Test 3 Review Answer Key

Dr. Graham-Squire, Spring 2013

1. Determine whether the sequence is convergent or divergent. If convergent, find the limit.

   (a) $a_n = \dfrac{n^3 + n^2 \cos n}{n^3}$

   **ANS: Converges to 1. Split it into two fractions and use comparison.

   (b) $b_n = \dfrac{\sqrt[3]{n}}{\ln n}$

   **ANS: Diverges. Use L'Hospital's rule.

2. Determine if the series is convergent or divergent. If it is convergent, find the limit. Make sure you state which convergence/divergence test you use (or if no test, then explain your reasoning).

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{5^{n+2}}$

   **ANS: Geometric series. Converges to 3/50.

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{2}{n(n+2)}$ Note: You may need to use a partial fraction decomposition.

   **ANS: Telescoping series. Converges to 3/2.

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{3n^2+8}{(10n+1)^2}$

   **ANS: The limit of the terms approaches $3/100 \neq 0$, so the series diverges by the test for divergence.

3. Test the series for convergence or divergence. If it converges, state whether it is absolutely convergent or not.

   (a) $\displaystyle\sum_{n=2}^{\infty} \frac{(-5)^n - 7}{4^n}$

   **ANS: Diverges. Split it into two fractions, one is geometric with $|r| > 1$ so it diverges (the other converges, but it does not matter).

   (b) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\sqrt[3]{n^4} + 10}{n^2}$

   **ANS: Converges by the alternating series test, but is not absolutely convergent (to see it is not absolutely convergent you can do a limit comparison with $1/n^{(2/3)}$, which is a divergent p-series).

   (c) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n^4}{3^n}$

   **ANS: Use the ratio test, will be absolutely convergent.

4. (a) Find the sum of $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^4}$ correct to four decimal places.

   **ANS: Use the alternating series remainder test to see that you only need to add up the first ten terms, that is $1 - \frac{1}{16} + \frac{1}{81} - \dots$.

   (b) How would you find the approximation if it was a 1 in the numerator instead of $(-1)^{n+1}$ (That is, how would you figure out how many terms you need to add up)?

   **ANS: You would need to use the integral remainder test and find for what value of k you have $\int_k^{\infty} \frac{1}{x^4}\, dx$ less than 0.0001.

5. Find the radius of convergence and interval of convergence of the series $\sum_{n=1}^{\infty} \frac{(x+2)^n}{n(4^n)}$

   **ANS: Use the ratio test to find that $R = 4$ and the interval is $[-6, 2)$.

6. Find a power series representation for $\frac{-2}{(1-x)^3}$ and its radius of convergence.

   **ANS: Integrate twice to get $-(1-x)^{-1}$, then write that as a power series and differentiate it twice to get a power series representation of $\sum_{n=2}^{\infty} n(n-1)x^{n-2}$ with radius of convergence equal to 1.